Macroeconomic costs of gender gaps in a model with entrepreneurship and household production: the case of Mexico

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Abstract

This paper uses the framework of Cuberes and Teignier (2017) to study the quantitative effects of gender gaps in entrepreneurship and workforce participation in Mexico. The focus on one specific country allows us to have detailed information on men and women’s participation in household production and their productivity in that sector. In line with our previous research, the occupational choice model predicts substantial losses in the country’s income per capita. Gender gaps in the Mexican labor market, especially in labor force participation, represent a 22% fall in total output. Market output drops by 26.5%, while household output experiences a five-fold increase. The presence of the large gap in labor force participations implies that it is important to introduce a household sector in the model to take into account the production that takes place outside the market sector.

JEL classification numbers: E2, J21, J24, O40.

Keywords: gender inequality, household production, factor misallocation, aggregate productivity.

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1 Introduction

Gender inequality is present in many socioeconomic indicators around the world in both developed and developing countries. Although recent decades have witnessed a significant drop in gender gaps in many countries, the prevalence of gender inequality is still high and it is present in several dimensions, including treatment in the labor market, education, political representation, and bargaining inside the household. In the labor market, for example, women typically receive lower wages, are underrepresented in many occupations, work fewer hours than men, and have less access to productive inputs. We also know that women typically carry out a much larger share of household chores than men.

Mexico is an interesting country to study gender inequality in the labor market and its impact on the macroeconomy. In Cuberes and Teignier (2016) we use data from the International Labor Organization to calculate that the gender gaps in labor force participation and employership. The two figures below show that Mexico is a very clear outlier among OECD countries.

Figure 1: Gender gaps in LFP across OECD countries

In this paper we calibrate the model in Cuberes and Teignier (2017) using Mexican data. Throughout the paper we compare our results to those of calibrating the model for the United States. We think using the U.S as the benchmark model is useful for two reasons. First, the two economies have very marked differences, both in terms of fundamentals and in terms of the role played by women in the labor market. Second, several of the parameters used to
calibrate the model are taken from US data, for which the data are much more reliable than in any other country.

As in our previous work, we find that the income losses associated with gender gaps in the labor market are substantial. In Mexico, these costs amount to about 22% of income per capita, almost twice as high as in the U.S. case (12.8%). An important finding is that most of the income loss of Mexico is generated by the extremely large gap in labor force participation. Since only 46 women participate in the labor market for every 100 men, the income losses associated with the LFP gap are huge (14% vs 4.7% in the US case). Measuring the household sector output in the model is important because there is a very large fraction of women not working in the labor market who can work in the household. The introduction of labor market gender gaps generates a five-fold increase in household production, much larger than in the US case. With respect to the entrepreneurship gender gap, in the case of Mexico, its role is dwarfed by the LFP gap.

The rest of the paper is organized as follows. In Section 2 we present the theoretical framework. We show the parameter values and the numerical results in Section 3, while we study the effects of technology change in household durables in Section 4. Section 5 concludes.

2 Theoretical framework

In this section, we present the theoretical framework used to generate the quantitative predictions of Section 3, which is an extension of the model proposed by Cuberes and Teignier (2016). The details of the model solution are presented in the Appendix.

2.1 Setup description

The economy we consider has two sectors (market and household) that produce an homogeneous good, as well as a continuum of agents, indexed by their skill level \( x \), who own one unit of time. Talent here should be interpreted more broadly than in Lucas (1978) or
Cuberes and Teignier (2016) since now it not only affects the entrepreneurs’ profits, but also the workers’ earnings. We assume the economy is closed, with an exogenous workforce of size \( P \). Skill-adjusted labor and capital are supplied by consumers to firms, in exchange for a wage rate per unit of skill, \( w \), and a capital rental rate, \( r \), respectively. These inputs are then combined by firms to produce a unique, homogeneous consumption. The stock of capital takes its steady-state value and, hence, its marginal product is equal to the depreciation rate plus the intertemporal discount factor.

Men choose to become either firm workers in the market sector, who earn the equilibrium wage rate \( w \) times their skill level \( x \), or entrepreneurs, who earn the profits generated by the firm they manage in the market sector. Women can also become workers or entrepreneurs but they also have the option of producing in the household sector. As in Lucas (1978) and Buera and Shin (2011), the production function of an employer is given by

\[
y(x) = x \left( k(x)^{\alpha} n(x)^{1-\alpha} \right)^{\eta},
\]

where \( x \) denotes the talent or productivity level of the employer, \( n(x) \) is the units of skill-adjusted labor hired by the employer, \( k(x) \) is the units of capital rented by the employer, and \( y(x) \) represents the units of output produced. The parameter \( \eta \in (0,1) \) measures the span of control of entrepreneurs and, since it is smaller than one, the entrepreneurial technology involves an element of diminishing returns. Since the price of the homogeneous good is normalized to one, employers’ profits are equal

\[
\pi(x) = y(x) - rk(x) - wn(x).
\]

On the other hand, an agent with talent \( x \) who chooses to become self-employed in the market sector operates a technology given by

\[
\tilde{y}(x) = \tau x \left( \tilde{k}(x)^{\alpha} \tilde{n}(x)^{1-\alpha} \right)^{\eta},
\]

where \( \tilde{k}(x) \) denotes the units of capital used and \( \tilde{y}(x) \) the units of output produced. \( \tilde{n}(x) = x \)

\(^{1}\text{In what follows we will refer to an entrepreneur as someone who works as either an employer or a self-employed.}\)
are the skill-adjusted labor units the self-employed agents works in his or her own firm.\footnote{The consumption good produced by the self-employed and the capital they use is the same as the one in the employers’ problem. However, it is convenient to denote them $\tilde{y}$ and $\tilde{k}$ to clarify the exposition.} The parameter $\tau$, which is calibrated to match the aggregate share of self-employed workers, captures the fact that self-employed agents have to spend some time on management tasks. Self-employed profits are equal to $\tilde{\pi}(x) = \tilde{y}(x) - r\tilde{k}(x)$.

Finally, women can also produce in the household sector, operating the following technology:

$$y_h = (Ak_h + Bn_h)^{\eta}, \quad (3)$$

where $k_h$ denotes the units of capital rent by the household sector and $n_h$ the units of time allocated to the household sector. Note that this production function can be seen as the perfect substitutes version of the one in equation (1), with the productivity parameters $A$ and $B$ being independent of the agent talent. Women choose $k_h$ and $n_h$ in order to maximize their total earnings, which are given by their market-sector plus their household sector earnings.\footnote{Arguably this is a unitary approach to the problem in the sense that a household in this model is effectively composed of only one person who can either be a man or a woman. A more realistic but complicated approach would recognize the importance of intra-household decisions as in Chiappori (1997). We leave this promising avenue for further research.} Specifically, when the opportunity cost of time is their market wage $wx$, women choose to allocate their unit of time in the household sector when $\frac{A}{B} < \frac{r}{wx}$, and they choose to allocate it to the market otherwise.\footnote{As explained in Appendix A, depending on the parameter values, women choosing to work at home may still want to rent some capital because their time endowment is limited. At the same time, there may be a group of women who allocate part of their time to the household sector and part of their time to the market sector.} Under this household production function, changes in the home technology parameter $A$ (which can be interpreted as an increase in the availability of home appliances or the consumer durable goods revolution mentioned in Greenwood et al., 2005) lead to a rise of female labor participation, as in the model by Greenwood et al. (2005) which is empirically assessed by Cavalcanti and Tavares (2008).

### 2.2 Frictionless Equilibrium

In equilibrium, employers choose the units of labor and capital they hire in order to
maximize their current profits, denoted by $\pi_e$, while self-employed workers choose the units of capital to rent in order to maximize their profits, denoted by $\pi_s$. Market workers earn a labor compensation equal to $wx$. Women also choose the units of capital to rent for the household-sector production and the fraction of their time they want to allocate to this sector. If they choose to become full-time household workers, they earn an income denoted by $\pi_{h0}$, while if they choose to become part-time household workers, they earn an income denoted by $\pi_{h1}$, which includes market-sector earnings plus household-sector earnings.

The first plot of Figure 2 displays the payoff of the three market occupations at each talent level and shows the optimal occupational choices in equilibrium for men. Men with the highest skill level (those with talent above $z_2$) become employers, whereas those with intermediate skill levels become self-employed. Finally, men with a level of talent lower than $z_1$ become market workers. The second plot of Figure 2 displays the slightly more complicated occupational map for women. As it was the case for men, women with talent above $z_2$ become employers, whereas those with talent between $z_1$ and $z_2$ choose to be self-employed. Women become market workers if their talent is between $z_{f0}$ and $z_1$. Women with talent below $z_{f0}$ allocate their time to the household sector production, either part time (between $z_{f00}$ and $z_{f0}$) or full time (below $z_{f0}$).\footnote{To be precise, $\pi_{h0}$ and $\pi_{h1}$ are defined here as the household production profits by household workers relative to market workers, who may also choose to engage in household production but using only capital.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The occupational map}
\end{figure}
In this economy, aggregate (market) production is the sum of output by male employers and male self-employed, as well as output by female employers and female self-employed:

\[ Y = N \left[ \int_{z_2}^{\infty} y(x) d\Gamma(x) + \int_{z_1}^{z_2} \bar{y}(x) d\Gamma(x) \right], \]

where \( \Gamma(x) \) denotes the talent cumulative density function, which, again, it is assumed to be the same for men and women. The first term inside the bracket represents the production by male and female employers, whereas the second one is the corresponding term by self-employed.

Total production in the economy, \( Y_T \), is the sum of market output \( Y \) and household output, \( Y_h \).

\[ Y_T = Y + Y_h. \]

\( Y_h \) is equal to household production by full-time household workers, \( y_{h0}^{00} \), plus household production by part-time household workers, \( y_{h1}^{01} \), plus household production by female market workers, \( y_h^1 \) (who use some capital in the household sector in order to produce there):

\[ Y_h = \frac{N}{2} \left( \int_{z_{00}}^{z_{00}'} y_{h0}^{00} d\Gamma(x) + \int_{z_{01}}^{z_{01}'} y_{h1}^{01}(x) d\Gamma(x) + \int_{z_0}^{z_0'} y_{h}^1 d\Gamma(x) \right). \]

### 2.3 Introducing gender gaps into the framework

The model assumes that women are identical to men in terms of their innate skills but they face exogenous constraints in their market-sector occupational choice. These frictions may reflect discrimination, or other demand factors, but they might also reflect differences in optimal choices of women, or other supply factors. In this sense, our estimated effects should be interpreted as the result of all the factors that make women behave differently than men in the labor market.
The first constraint we impose is that females face a probability $\mu$ of being “allowed” to be an employer and a probability $1 - \mu$ of being excluded from employership. Out of the group of women not allowed to be employers, some have the possibility of becoming self-employed while the rest are also excluded from self-employment. In particular, women excluded from employership have a probability $\mu_o$ of being allowed to be self-employed and a probability $(1 - \mu_o)$ of not being allowed to be self-employed. As a result a fraction $(1 - \mu)(1 - \mu_o)$ of women are shut out from entrepreneurship, i.e. both employership and self-employment, and can only become workers. Appendix B shows a graphical representation of the occupational choice of women taking into account the constraints just described. Finally, the third friction we introduce is that only a fraction $\lambda$ of women are allowed to participate in the labor market, while a fraction $(1 - \lambda)$ of randomly selected women are excluded from all the possible occupations in the labor market. In this setup, women who do not participate in the formal labor market become full-time workers in the household sector and, hence, the estimated aggregate income loss due to the $\lambda$ gender gap depends on the difference between the market participants earnings and the household-sector earnings.

The effects of the entrepreneurship gaps, $\mu$ and $\mu_o$, are illustrated in Figure 3 for the case without part-time workers. When some women are excluded from entrepreneurship, the supply of market workers increases, leading to a fall in the wage rate and a rise in the employers’ profit function. This makes both $z_1$ and $z_2$ fall, implying a lower average talent of entrepreneurs and a lower firm productivity. The capital stock adjusts downwards to keep its marginal product equal to the depreciation rate plus the intertemporal discount factor. Moreover, in the case of women, there is a rise in $z_{0f}^f$, implying that the number of workers in the market-sector falls and the number of workers in the household sector rises. As a result of

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6 Again, this constraint may represent either demand barriers, supply choices, or a combination of the two. 7 Note that, in this setup, we are not allowing for the possibility of women being excluded from self-employment but not from employership, since we think that whichever are the barriers women face to become self-employed, they should apply even more strongly to become an employer. In terms of the parameters of the model, if $\mu = 1$, then the value of $\mu_o$ does not affect the occupational choices of women.

8 We say that women excluded from the labor force are randomly selected because the talent of these excluded women is drawn from the same distribution as the rest of the population.
all these effects, market-sector output decreases. If part-time work was also considered, the fall in wages would lead to a rise of both $z_{00}^f$ and $z_0^f$, implying also a fall in female market work.

**Figure 3:** Qualitative effects of entrepreneurship gender gaps

The effects of the labor force participation gap, $\lambda$, are more straightforward. When some women get excluded from the market sector, they become household-sector workers, leading to a fall in the market-sector labor and a rise in the home-sector labor. As before, the capital stock adjusts downwards to keep its marginal product equal to the depreciation rate plus the intertemporal discount factor. These effects clearly reduce total output from the market sector, but they are likely to slightly increase output per worker because the household-sector capital demand falls and, thus, the market sector capital-to-labor ratio increases.

### 3 Numerical results

#### 3.1 Talent Distribution and Model Parametrization

To simulate the model, we use a Pareto function for the talent distribution, as in Lucas (1978) and Buera et al. (2011). The cumulative distribution of talent is, hence, given by

$$\Gamma (x) = 1 - B^\rho x^{-\rho}, \ x \geq 0,$$  

(4)
where $\rho, B > 0$.

The values used for the model parameters are showed in Table 1. The parameter $B$ of the talent distribution is normalized to 1, while the parameter $\eta$ is set to 0.79 as in Buera and Shin (2011). The capital-output elasticity parameter $\alpha$ is set to 0.114 in order to match the 30% capital income share observed in the data. The parameters ($\rho, \tau, A, B$) are estimated to match four different moments of the Mexican data. First, the fraction of employers in the labor force (which is 4.4%), second, the fraction of self-employed workers in the labor force (which is 22.3%), third, the household sector productivity relative to the market-sector one (which is 0.26), and, fourth, the gap between the share of female part-time workers and the share of male part-time workers (which is 12.4%). Compared to the United States case, we observe that Mexico has a slightly larger share of employers (4.4% vs. 3.6), a much larger share of self-employed workers (22.3% vs 6.5%), and a similar relative household productivity and part-time gap.

Data on employment status and working hours is obtained from the National Survey of Occupation and Employment (ENOE) made public by the National Statistics Institute (INEGI). The National Statistics Institute (INEGI) compiles satellite accounts on non-remunerated household work. The data show that this type of work amounted in 2017 to 5.1 billion pesos (0.25 billion US dollars), or about 23% of Mexico’s GDP. The estimation of this satellite account is based on two inputs: 1) A measure of time spent on unpaid work. This is approximated through the number of hours of unpaid work and the identification of the individuals who perform it, both indicators are taken from the National Time Use Survey; and, 2) The cost per hour spent on unpaid care and domestic work, estimated from the National Occupation and Employment Survey providing gross values from average earnings by economic activity, according to the North American Industry Classification System (NAICS). The activities included for this estimation are those household’s activities defined as productive, if

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9Buera and Shin (2011) choose $\eta$ to match the top five percent income share in the U.S., which is 30%. This is a reasonable approximation given that the top earners are entrepreneurs both in the model and the U.S. data.

10 Entrepreneurs’ profits are considered capital income, thus we set $\alpha \eta + (1 - \eta)$ equal to 30%.
can be delegated to somebody else or provide a product or service that can be exchanged in the market, like provision of: food, cleaning and maintenance of a dwelling, cleaning and care of clothes and shoes, shopping and household management, care and support, community services and volunteer work.\textsuperscript{11}

The values of the country-specific gender gaps ($\mu, \mu_o, \lambda$) are computed to simultaneously match the female-to-male ratio of employers, self-employed workers, and labor market participation in each country. After matching these moments, we obtain that the value of the employership gender gap, $1 - \mu$, is 0.6 (very similar to the U.S. one), while the self-employment gender gap, $(1 - \mu) (1 - \mu_o)$ is equal to 0.08 (compared to 0.41 in the U.S.), and the labor force gender gap, $1 - \lambda$, is 0.44 (compared to 0.14 in the U.S.).

### 3.2 Numerical results

The numerical results for Mexico are summarized in Table 2, which shows that gender gaps lead to a fall in total output (market plus household) is much larger in Mexico than in the United States (22\% vs. 12.7\%). In Mexico, there is an almost five-fold rise in household sector production due to the presence of gender gaps (487\% in Mexico vs. 6.5\%) but this does not compensate for the fact that the fall in market output is much larger in Mexico (26.5\% vs. 17.3\%). The effects of the entrepreneurship gender gaps on market output, however, are larger in the United States, the reason being that the fall in female market sector hours due

\textsuperscript{11}Due to the very nature of the non-remunerated activities, some degree of measurement error should be assumed.

### Table 1: Common parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.79</td>
<td>From Buera and Shin (2011)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>7.35</td>
<td>To match the employer’s share in Mexico</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.697</td>
<td>To match the self-employed’s share in Mexico</td>
</tr>
<tr>
<td>$A_n$</td>
<td>0.307</td>
<td>To match the value of household output</td>
</tr>
<tr>
<td>$A_k$</td>
<td>0.055</td>
<td>To match the share of female part-time workers</td>
</tr>
</tbody>
</table>
Table 2: Average losses due to the gender gaps in Europe and the United States

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall in market output due to entrepreneurship gaps</td>
<td>12.47</td>
<td>9.44</td>
</tr>
<tr>
<td>Fall in market output due to all gender gaps</td>
<td>17.26</td>
<td>26.5</td>
</tr>
<tr>
<td>Fall in household output due to all gender gaps</td>
<td>-6.48</td>
<td>-487.27</td>
</tr>
<tr>
<td>Fall in total output due to all gender gaps</td>
<td>12.68</td>
<td>22.01</td>
</tr>
<tr>
<td>Fall in female mkt hours due to entrepreneurship gaps</td>
<td>11.87</td>
<td>0.11</td>
</tr>
<tr>
<td>Fall in female mkt hours due to all gender gaps</td>
<td>23.65</td>
<td>44.19</td>
</tr>
</tbody>
</table>

to the entrepreneurship gender gaps is significantly smaller in Mexico (0.11% vs 11.9%)

4 Conclusion

This paper uses a general equilibrium, occupational choice model with a household sector to examine the quantitative effects of gender gaps in entrepreneurship and workforce participation in Mexico. Our main finding is that the presence of gender gaps generates large losses in aggregate income. The introduction of a household sector in the model is important because it allows women not participating in the labor market to work at home. Because labor force participation gender gaps in Mexico are huge compared to entrepreneurship gaps, the main consequence of considering the household sector is the gain in household production generated by the LFP gaps. This is a striking difference with the case of the US, where the entrepreneurship gap also generates a large rise in female household hours and, as a result, an increase in household sector output.


A Model details

A.1 Agents’ optimization

A.1.1 Employers

Employers choose the units of labor and capital they hire in order to maximize their current
profits $\pi$.

$$\max_{k,n} \left\{ x \left( k^\alpha n^{1-\alpha} \right) - r k - w n \right\},$$

The optimal number of workers and capital stock, $n(x)$ and $k(x)$ respectively, depend positively on the productivity level $x$, as equations (5) and (6) show:

$$n(x) = \left( x \eta (1 - \alpha) \left( \frac{\alpha}{1 - \alpha} \right)^{\alpha \eta w^{\eta - 1}} \right)^{1/(1 - \eta)},$$

$$k(x) = \left( x \eta \alpha \left( \frac{1 - \alpha}{\alpha} \right)^{\eta (1 - \alpha)} \frac{r^{\eta (1 - \alpha) - 1}}{w^{\eta (1 - \alpha)}} \right)^{1/(1 - \eta)}.$$

A.1.2 Self-employed

When we solve for the problem of a self-employed agent with talent $x$ who wishes to maximize his or her profits,

$$\max_k \left\{ x k(x)^{\alpha \eta} - r k \right\},$$

we find

$$\tilde{k}(x) = \left( \frac{\tau x \alpha \eta}{r} \right)^{\frac{1}{1 - \alpha \eta}}.$$

A.1.3 Household production

Women can get extra earnings from household production, hence they choose the household units of capital $k_h$ and labor $n_h$ in order to maximize their total earnings, which are given by their market-sector plus their household sector earnings. Specifically, when their optimal occupational choice in the market is to become a worker, their optimization problem is

$$\max_{k_h, n_h} \left\{ (A k_h + B n_h)^\eta + wx (1 - n_h) \right\},$$
with \( n_h \in [0, 1] \) and \( k_h \geq 0 \). As a result, when \( \frac{A}{B} > \frac{r}{w_x} \), women choose to allocate all their time to the market sector and rent \( k_h^1 \equiv \left( \frac{n A^\eta}{r} \right)^{\frac{1}{1-\eta}} \) units of capital. When \( \frac{A}{B} < \frac{r}{w_x} \), on the other hand, women allocate at least part of their time endowment to the household sector. In particular, their optimal time allocation to the household sector is \( n_h^0 \equiv \min \left\{ 1, \left( \frac{n B^\eta}{w_x} \right)^{\frac{1}{1-\eta}} \right\} \), which implies that some women with high market productivity may choose to allocate part of their time to the household sector and part of their time to the market sector. Women supplying all their labor to the market sector choose to rent \( k_h^0 \equiv \max \left\{ 0, \left( \frac{n A^\eta}{r} \right)^{\frac{1}{1-\eta}} - \frac{B}{A} \right\} \) units of capital.

In other words, when \( \frac{r B^{1-\eta}}{n A} < 1 \), women choose their labor allocation as follows:

\[
n_h = \begin{cases} 
0 & \text{if } x > \frac{B}{A} \frac{r}{w} \\
1 & \text{otherwise}
\end{cases}
\]  

(8)

and their units of capital used in the household sector are equal to

\[
k_h = \begin{cases} 
\left( \frac{n A^\eta}{r} \right)^{\frac{1}{1-\eta}} & \text{if } x > \frac{B}{A} \frac{r}{w} \\
\left( \frac{n A^\eta}{r} \right)^{\frac{1}{1-\eta}} - \frac{B}{A} & \text{otherwise}
\end{cases}
\]

(9)

producing the following units of output:

\[
y_h = \left( \frac{n A^\eta}{r} \right)^{\frac{2}{1-\eta}}
\]

(10)

in both cases.

\(^{12}\)Note that if a woman is an employer or a self-employed, it will never be optimal for her to spend some time in household production.
On the other hand, when \( \frac{rB^{1-\eta}}{A^{\eta}} > 1 \), women choose their labor allocation as follows:

\[
n_h = \begin{cases} 
0 & \text{if } x > \frac{B}{A} \frac{r}{w} \\
\left( \frac{\eta B^\eta}{w} \right)^{\frac{1}{1-\eta}} & \text{if } \frac{\eta B^\eta}{w} < x < \frac{B}{A} \frac{r}{w} \\
1 & \text{if } x < \frac{\eta B^\eta}{w}
\end{cases}
\]  

(11)

and their units of capital used in the household sector are equal to

\[
k_h = \begin{cases} 
\left( \frac{\eta A^\eta}{r} \right)^{\frac{1}{1-\eta}} & \text{if } x > \frac{B}{A} \frac{r}{w} \\
0 & \text{otherwise}
\end{cases}
\]  

(12)

producing the following units of output:

\[
y_h = \begin{cases} 
\left( \frac{\eta A}{r} \right)^{\frac{\eta}{1-\eta}} & \text{if } x > \frac{B}{A} \frac{r}{w} \\
\left( \frac{\eta B^\eta}{w} \right)^{\frac{\eta}{1-\eta}} & \text{if } \frac{\eta B^\eta}{w} < x < \frac{B}{A} \frac{r}{w} \\
B^\eta & \text{if } x < \frac{\eta B^\eta}{w}
\end{cases}
\]  

(13)

A.1.4 Occupational choice

Figure (2) displays the shape of the profit functions of employers \( (\pi_e(x)) \) and self-employed \( (\pi_s(x)) \) along with wage function earned by employees and the female household workers extra earning as a function of talent \( x \).13 The figure also shows the relevant talent cutoffs for the occupational choices. Here we present the equations that define the three thresholds. The threshold, \( z_1 \), determines the earnings such that agents are indifferent between becoming workers or self-employed and it is given by

\[
wz_1 = \tau z_1 \tilde{k} (z_1)^{\alpha \eta} - r \tilde{k} (z_1) .
\]  

(14)

13In order to construct this figure we are implicitly using parameter values such all occupations are chosen in equilibrium and that part-time work is not optimal.
If \( x \leq z_1 \) agents choose to become workers, while if \( x > z_1 \) they become self-employed or employers. The cutoff, \( z_2 \), on the other hand, determines the choice between being a self-employed or an employer and it is given by

\[
\tau z_2 \bar{z}(z_2)^{\alpha \eta} - \bar{r}k(z_2) = z_2x \left( k(z_2)^{\alpha} n(z_2)^{1-\alpha} \right)^{\eta} - rk(z_2) - wn(z_2)
\]  

so that if \( x > z_2 \) an agent wants to become an employer.

Finally, the cutoff \( z_f^{f} \), defines the talent level at which women are indifferent between being household workers, who only get earnings from their household production, and market workers, who get wage income plus household income from the household capital production. Specifically, when \( \frac{rB^1 - \eta}{\eta A} < 1 \), household workers get earnings \( \left( \frac{nA}{r} \right)^{\frac{\eta}{1-\eta}} - r \left( \left( \frac{nA}{r} \right)^{\frac{1}{1-\eta}} - \frac{B}{A} \right) \), while market workers get their wage income plus household earnings equal to \( \left( \frac{nA}{r} \right)^{\frac{\eta}{1-\eta}} - r \left( \frac{nA}{r} \right)^{\frac{1}{1-\eta}} \). Hence, the difference between the household sector earnings is equal to \( r \frac{B}{A} \) and the talent threshold \( z_f^{f} \) is defined as

\[
r \frac{B}{A} = wz_f^{f}.
\]  

Therefore, if their talent is below \( z_f^{f} \), women maximize their earnings as household workers, while above \( z_f^{f} \) their earnings are maximized as market workers.

When \( \frac{rB^1 - \eta}{\eta A} > 1 \), on the other hand, there are some women working full time in the household sector, some working part-time in the household sector and part-time in the market sector, and some other women working full time in the market sector. Women with ability below \( z_{00}^{f} \), where \( z_{00}^{f} \equiv \frac{B^0}{w} \), choose to work full time in the household sector, and earn \( B^0 \). Women with ability between \( z_{00}^{f} \) and \( z_0^{f} \), where \( z_0^{f} \) is defined in equation (16), choose to allocate part of their time to the market and part of their time to the household. Their total earnings are \( \left( \frac{nB}{wx} \right)^{\frac{\eta}{1-\eta}} \) from the household production plus \( wx \left( 1 - \left( \frac{nB}{wx} \right)^{\frac{1}{1-\eta}} \right) \) from the market sector, compared to total earnings of \( wx + \left( \frac{nA}{r} \right)^{\frac{\eta}{1-\eta}} - r \left( \frac{nA}{r} \right)^{\frac{1}{1-\eta}} \) by female workers.

When \( \frac{rB^1 - \eta}{\eta A} > 1 \) women have actually five occupational choices, since some choose to work part time in the market and part time in the household sector. In this case, the earning
functions are defined as
\[
\pi_{h}^{00} \equiv B^{n} - (1 - \eta) \left( \frac{\eta A}{r} \right)^{\frac{n}{\eta}}
\]
and
\[
\pi_{h}^{01} \equiv w x + (1 - \eta) \left( \frac{\eta B}{w x} \right)^{\frac{n}{\eta}} - \left( \frac{\eta A}{r} \right)^{\frac{n}{\eta}}
\]
which correspond to the household workers earnings minus the household production earnings of female market workers.

A.2 Competitive Equilibrium in a model with household sector

We assume that women represent half of the population in the economy and that there is no unemployment. Moreover, any agent in the economy can potentially participate in the labor market, except for the restrictions on women described above. Under these assumptions, in equilibrium, the total demand of capital by employers and self-employed must be equal to the aggregate capital endowment (in per capita terms), \( k \):

\[
k = \frac{1}{2} \left[ \int_{z_{2}}^{\infty} k(x) d\Gamma(x) + \int_{z_{1}}^{z_{2}} \tilde{k}(x) d\Gamma(x) \right] + \frac{\lambda}{2} \left[ \int_{z_{2}}^{\infty} \mu k(x) d\Gamma(x) + \int_{z_{1}}^{z_{2}} (\mu + (1 - \mu)\mu_{0}) \tilde{k}(x) d\Gamma(x) + \int_{z_{2}}^{\infty} (1 - \mu)\mu_{0} \tilde{k}(x) d\Gamma(x) \right] + \frac{1 - \lambda}{2} \int_{z_{2}}^{\infty} k^{0}_{h} d\Gamma(x).
\]  (17)

The first line of equation (17) is the demand for capital by men, while the two lower lines are the women’s demand for capital. The demand for capital by male-run firms has two components: the first one represents the capital demand by employers, while the second represents the demand by self-employed.

The demand of capital by women has six components, the first three corresponding to the market-sector firms run by women and the last three corresponding to the household-sector capital. The first one represents the capital demand by female employers, i.e. those with enough ability to be employers and who are allowed to be so, while the second term represent
the capital demand by women who have the right ability to be self-employed. The third term shows the capital demand by women who become self-employed because they are excluded from employership. The fourth term corresponds to the household-sector capital demand by women who choose to be household-sector workers, the fifth is the household-sector capital demanded by women supplying the entire labor supply to the market sector, and the last term is the household-sector capital demand by women who work in the household-sector because they are not allowed to work in the market sector.

Similarly, the labor market-clearing condition is given by

$$1 \frac{1}{2} \left[ \int_{z_2}^{\infty} n(x) d\Gamma(x) \right] + \lambda \frac{1}{2} \left[ \int_{z_2}^{\infty} \mu(x) n(x) d\Gamma(x) \right] =$$

$$\frac{1}{2} \int_{B}^{z_f} xd\Gamma(x) + \lambda \frac{1}{2} \left[ \int_{z_0}^{z_f} xd\Gamma(x) + \int_{z_1}^{\infty} (1 - \mu)(1 - \mu_0)) xd\Gamma(x) + \int_{B}^{z_f} x (1 - n^0_h(x)) d\Gamma(x) \right],$$

where the first line represents the skill-adjusted aggregate labor demand and the second line represents the skill-adjusted aggregate labor supply in the market sector. The aggregate labor demand is equal to the male employers demand (first term) and the female employers demand (second term), i.e. those women with enough ability to be employers who are allowed to choose their occupation freely. The aggregate labor supply is equal to the male workers supply (first term in second line) plus the female workers supply (second, third, and fourth term in second line). The female workers supply is given by the skill-adjusted labor of women who, given their talent, choose to be full-time workers, plus that of women who have enough ability to be employers or self-employed but are excluded from both occupations. Finally, some women working in the household sector may also choose to be part-time workers in the market sector.

A competitive equilibrium in this economy is a set of cutoff levels \((z_{00}^f, z_0^f, z_1, z_2)\), a set of quantities \([n(x), n^0_h(x), k(x), \tilde{k}(x), k^0_h, \tilde{k}^1_h], \forall x\), and prices \((w, r)\) such that entrepreneurs choose the amount of capital and labor to maximize their profits, and labor and capital markets clear.
B Women occupational choice map

- (B, z₀)

  - (z₀, ∞)
    - Market participants
    - (z₁, ∞)
      - Can be either employers or self-employed
      - μ
    - (z₀, z₁)
      - Cannot be employers
      - Become workers
      - 1 − μ
    - (z₁, z₂)
      - Can be self-employed
      - Become self-employed
      - μ₀
    - (z₂, ∞)
      - Become employers

- (z₀, ∞)

  - Household workers

  - (z₀, z₁)
    - Become workers
    - 1 − μ₀
    - Cannot be self-employed
    - Become workers